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A new model of intergrain interaction in dusty plasmas

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Abstract

Of interest in modern plasma physics are investigations of dusty plasmas encountered in a variety of experiments, applications and natural phenomena as well. Dust grains are known to form a crystalline structure that poses a question of how attractive forces are generated in a particular dusty plasma. In this paper, a new model of intergrain interaction is proposed. In most works on dusty plasmas, the plasma medium itself is considered to be in an ideal state of matter when the interaction between particles, i.e., electrons and ions, is negligible. Herein this assumption is not implied and, thus, the density–response formalism with the local-field correction is used to account for the plasma screening in the longitudinal dielectric function. Then, a new model of intergrain interaction is defined and it is shown that a molecule-like potential with an energy well appears. Such a model might be appropriate for cryogenic dusty plasma experiments recently reported.

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In recent years, a certain boost in studying various properties of dusty plasmas has been witnessed in the literature [1, 2]. It is mostly due to the fact that besides electrons and ions such a plasma contains macroscopic particles, called grains, whose behaviour is easily visualized in experiment. This makes it possible to check many concepts and approaches to strongly coupled systems against the experimental data already available. One of the important problems set by those experiments is how the crystalline structure formation is facilitated in a particular plasma experiment. The answer to it implies that in spite of the like electric charge of grains there must be an attractive force between them. This report is devoted to a new mechanism of intergrain attraction caused by the nonideality of the plasma medium.

In the following, the system of interest is an aggregate of dust grains with the electric charge $Z_{\mathrm{d}}e$ and the number density n_{d} that are immersed in a one-component plasma of ions (or electrons, it does not really matter) with the electric charge e and the number density n. Electrons are supposed to constitute a uniform background but in what follows one will see how this simplification might be avoided by incorporating a two-component plasma in full.

The first parameter relevant to the description of the dust component is the dust coupling parameter

$$\Gamma_{\mathrm{d}} = \frac{Z_{\mathrm{d}}^2 e^2}{a_{\mathrm{d}} k_{\mathrm{B}} T}, \tag{1}$$

where $a_{\mathrm{d}} = (3/4\pi n_{\mathrm{d}})^{1/3}$ refers to the mean intergrain spacing, k_{B} is the Boltzmann constant and T denotes the medium temperature. In virtue of the high electric charge of dust grains, the dust coupling parameter can range from several tens to several hundreds or even thousands.

The other dimensionless parameter that relates the dust component to the plasma medium is called a screening parameter and is defined as

$$\kappa = \frac{a_{\mathrm{d}}}{\lambda_{\mathrm{D}}}, \tag{2}$$

where $\lambda_{\mathrm{D}} = (k_{\mathrm{B}} T/4\pi n e^2)^{1/2}$ denotes the Debye screening radius of the one-component plasma.

It is worthwhile mentioning here that due to the shielding of the electric field the real coupling of dust grains in the medium is approximately equal to $\Gamma_{\mathrm{d}} \exp(-\kappa)$.

If the one-component plasma may be considered an ideal gas, two parameters in (1) and (2) are quite sufficient to completely characterize the state of the dusty plasma. Otherwise, it is necessary to introduce another dimensionless parameter

$$\Gamma = \frac{e^2}{a k_{\mathrm{B}} T}, \tag{3}$$

which is then called the Coulomb coupling parameter. Here $a = (3/4\pi n)^{1/3}$ designates the mean distance between ions.

Thus, the three parameters (1)–(3) are quite enough to achieve the complete theoretical description of the system under consideration.

To determine the dusty plasma characteristics, it is essential to know the form of the interaction potential between two particular grains. Hereafter, the dielectric medium approximation (DMA) is utilized which means that a certain pair of grains is placed in a plasma medium that plays the role of a polarizable background generating the screening phenomenon. Consequently, the pairwise interaction potential between two grains is written as follows:

$$\Phi(r) = \frac{1}{(2\pi)^3} \int \frac{\tilde{\varphi}(k)}{\varepsilon(k,0)} \exp(\mathrm{i}\mathbf{kr}). \tag{4}$$

Here $\tilde{\varphi}(k) = 4\pi Z_{\mathrm{d}}^2 e^2 \big/ k^2$ represents the Fourier transform of the Coulomb intergrain interaction potential in vacuum and $\varepsilon(k,0)$ stands for the static dielectric function of the plasma medium.

In each particular case, the interaction potential in (4) depends on the choice of the static dielectric function which is, in turn, associated with the physical conditions in the plasma. For instance, if the plasma is in an ideal gas state the most appropriate form of the static dielectric function is the random phase approximation (RPA) which reads

$$\varepsilon_{\mathrm{RPA}}(k,0) = 1 + \frac{1}{k^2 \lambda_{\mathrm{D}}^2}. \tag{5}$$

Substitution of (5) into (4) gives rise to the Debye–Hückel (Yukawa) potential with the ordinary exponential screening

$$\Phi(r) = \frac{Z_{\mathrm{d}}^2 e^2}{r} \exp\left(-\frac{r}{\lambda_{\mathrm{D}}}\right). \tag{6}$$

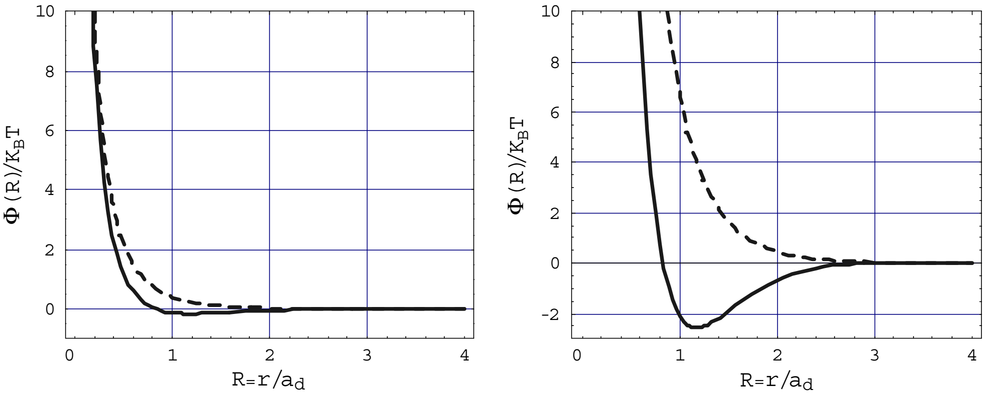

Figure 1. The intergrain interaction potential at various parameters. Left-hand side: $\Gamma = 3$, $\kappa = 2$ and $\Gamma_d = 3$; right-hand side: $\Gamma = 3$, $\kappa = 2$ and $\Gamma_d = 50$. Dashed line: Debye–Hückel potential (6); solid line: potential (4) with (7).

The interaction between the background plasma and dust grains is omitted in (4) but note that the Yukawa potential (6) is successfully used in both theoretical approaches and simulations which validates the dielectric medium approximation introduced above.

In the event that the plasma medium is moderately or even strongly coupled, the local-field correction (LFC) can be introduced into the density–response formalism and the static dielectric function acquires the following form [3]:

$$\varepsilon_{\mathrm{LFC}}(k,0) = 1 + \frac{1}{k^2\lambda_{\mathrm{D}}^2 - G(k)}. \tag{7}$$

$G(k)$ represents the so-called static local-field function and is expressed as [3]

$$G(k) = 1 + \frac{k_{\mathrm{B}}T\tilde{C}(k)}{\tilde{\varphi}(k)}, \tag{8}$$

where $\tilde{C}(k)$ designates the Fourier transform of the direct correlation function $C(r)$.

Of course, the very simplest way of obtaining the direct correlation function is brought by the set of the hyper-netted chain (HNC) equations that has been solved numerically for the one-component plasma. In this way, the Fourier transform of the direct correlation function has been obtained and it has allowed one, with the aid of (8), to derive the static dielectric function within LFC. Ultimately, all just mentioned has delivered a new form of the intergrain interaction potential (4) for which numerical results are presented in figure 1. This figure clearly shows that in the case of the nonideal plasma medium the intergrain interaction potential has a minimum at a distance of about mean intergrain spacing and it also has a long-range attractive part in the tail even though the grains are of like electric charge. Such an attractive force can simply facilitate the dust molecule formation and, as shown below, may result in the crystalline structure of dust particles at rather small values of the dust coupling parameter. Note that the depth of the potential well increases when the dust coupling parameter grows since the potential in (4) is directly proportional to the dust coupling parameter. According to the remark made above, the real dust coupling in figure 1 is equal to 0.41 and 6.77, respectively.

It is now rather timely to elucidate some physical reasons behind the non-monotonic behaviour of the intergrain potential. Since the screening is produced by the plasma, the appearance of the minimum in the potential curve is imperatively related to the physical

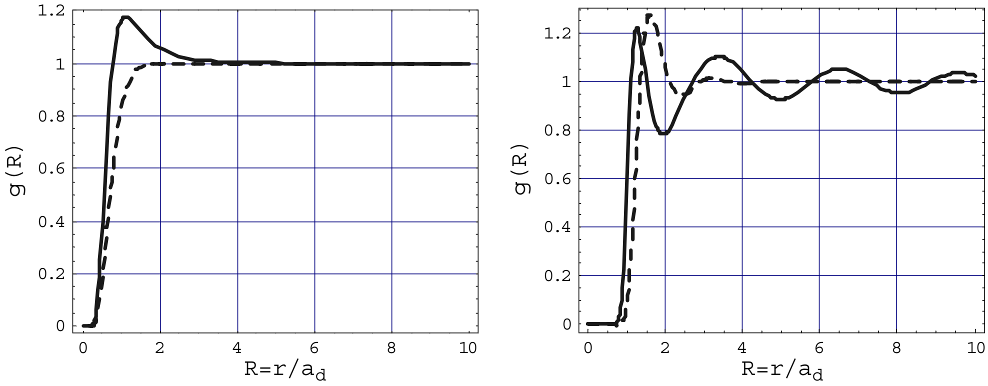

Figure 2. The radial distribution function at various parameters calculated via HNC. Left-hand side: $\Gamma = 3, \kappa = 2$ and $\Gamma_d = 3$; right-hand side: $\Gamma = 3, \kappa = 2$ and $\Gamma_d = 50$. Dashed line: Debye–Hückel potential (6) (cf [4]); solid line: potential (4) with (7).

conditions in the medium. Let a dust grain be negatively charged. Then, it attracts ions which form a cloud around it and, together with the background electrons, shield the electric field of the dust grain. Such a picture remains in power for both ideal gas and strongly coupled states of the plasma. In the latter case, though, there is short-range order in the ion distribution, i.e. there is a maximum in the probability density of finding two ions at a certain distance from each other. Thus, that ionic cloud around the dust grain condenses in comparison with the ideal gas state which finally results in the non-monotonic behaviour of the intergrain potential.

Since the intergrain interaction potential has been determined above, the correlation phenomena in dusty plasmas can immediately be studied. To do so, the HNC equation is again solved and the numerical results are presented in figure 2. It is seen that only the short-range order formation is observed at rather small values of the dust coupling parameter whereas at larger values the crystalline structure with a number of peaks occurs. Note that such a crystalline structure (long-range order) is shaped at relatively small values of the dust coupling parameter in comparison with the case when the plasma medium is considered an ideal gas of ions.

The only important question remaining is where in practice the nonideality in plasma particle interaction might play a significant role and, thus, in what physical situations the results described above could be observed experimentally. It has been reported recently that the crystalline structure formation is made much easier for cryogenic dusty plasmas [5]. The cryogenic dusty plasma experiment is an ordinary set-up for the radio-frequency glow discharge in which the gas is cooled by liquid nitrogen or helium. Unfortunately, the plasma parameters were not strictly measured but it seems reasonable to assume the ion concentration $n = 10^{10}$ sm^{-3} with the temperature $T = 3$ K. The Coulomb coupling parameter is then evaluated in (1) as $\Gamma \approx 2$ and, consequently, the plasma medium may be considered at least moderately coupled.

It is obvious that the approach presented above has its own merits and demerits. First, the final size of grains is not taken into account and, thus, there is no strict possibility to account for the charging process of dust particles. Second, the intergrain potential obtained is not valid at very close distances when the non-linearity effects become essential [6] but it can still be used in theory and simulations to be made.

On the basis of the above stated results, it is possible to make a few important inferences: (i) a new model of intergrain interaction taking into account the nonideality of the plasma medium has been proposed; (ii) the potential curve is somewhat reminiscent of the molecule potential with the energy well whose depth is proportional to the dust grain coupling; (iii) correlations in such a dusty plasma exhibit the crystalline structure formation (long-range order) at much smaller magnitudes of the dust coupling parameter in comparison with the situation when the plasma medium is considered an ideal state of matter.

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